Body-force effect on the lateral movement of cellular flames at low Lewis numbers

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The body-force effect on the lateral movement of cellular flames is studied by unsteady calculations of reactive flows at low Lewis numbers. We employ the compressible Navier-Stokes equation including chemical reaction to take account of the hydrodynamic effect caused by thermal expansion. A sinusoidal disturbance with the linearly most unstable wavelength is superimposed on a plane flame to simulate the formation of a cellular flame. The superimposed disturbance grows initially with time, and then the flame front changes from a sinusoidal to a cellular shape. After the cell formation, the cellular flame moves laterally at Lewis numbers lower than unity. The reason is that the diffusive-thermal effect, and the nonlinear effect of the flame front, play a primary role in the appearance of the lateral movement of cells. The body-force effect has a great influence on the lateral velocity of cells. When flames are propagated upward, the lateral velocity decreases as the acceleration increases, even though the body-force effect has a destabilizing influence. When flames are propagated downward, on the other hand, the lateral velocity takes a maximum value at the specific acceleration and decreases with an increase in acceleration. The dependence of lateral velocity on the acceleration is due to the augmentation and diminution in maximum flame temperature and to the broadness and narrowness of a high-temperature region behind a convex flame front.

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I. INTRODUCTION

It is well-known that hydrodynamic, body-force, and diffusive-thermal effects are of vital importance to the intrinsic instability of premixed flames [1–3]. Appearances of cellular flames and of the unsteady motion of flame fronts are due primarily to these effects.

The hydrodynamic effect caused by thermal expansion has a destabilizing influence on the flame front [4,5]. This type of instability is called Darrieus-Landau instability. The hydrodynamic effect is essential to the intrinsic instability of all premixed flames, since combustible gases expand thermally as they pass through the flame front. Thus, we have to take account of this effect in the study of the flame instability.

The body-force effect occurs when above and below fluids have different densities. We know that a light fluid beneath a heavy fluid is unstable, which is called Rayleigh-Taylor instability. The density of gases is drastically changed through the flame front, so that the body-force effect has an influence on the instability of gaseous flames anywhere on earth, i.e., it has a destabilizing (stabilizing) influence on upward (downward) propagating flames [6–8]. It is experimentally shown that the dynamics of cellular flames is greatly affected by the body-force effect [9,10]. In addition, this effect becomes more conspicuous as the burning velocity decreases [3]. Thus, we need to consider the body-force effect on instability particularly in slowly propagating flames.

The diffusive-thermal effect originates in the interaction of diffusion and heat-conduction processes [11–13]. Its performance depends on the Lewis number of the deficient component. When the Lewis number is lower (higher) than unity, the flame front becomes unstable (stable). The typical example of the diffusive-thermal effect is the formation of cellular flames. Premixed flames are more likely to exhibit a cellular shape at lean mixtures of light fuels, i.e., hydrogen and methane, with air and at rich mixtures of heavy fuels, e.g., propane and butane. The reason is that the formation of a cellular shape is governed by intrinsic instability, especially, diffusive-thermal instability.

Cellular flames formed by the above-mentioned effects are steady or unsteady, depending on the mixture component [6,14–17]. To investigate the temporal motion of flame fronts at low Lewis numbers, nonlinear analyses based on the diffusive-thermal model equation were performed, and it was reported that traveling waves along the flame front arise at sufficiently low Lewis numbers [18,19]. However, the obtained results are valid only for flames with a sufficiently small heat release, since the constant-density approximation is used, i.e., the hydrodynamic effect is disregarded in the model equation. In general, flames have a considerably large heat release, and the hydrodynamic effect has a great influence on the temporal motion of flame fronts. Thus, it is necessary to employ the compressible Navier-Stokes equation in numerical simulation to take account of the hydrodynamic effect.

Based on the compressible Navier-Stokes equation, several calculations on unsteady reactive flows were performed to investigate the flame instability and the formation of cellular flames [20–24]. The hydrodynamic effect is one of the most important factors in cellular-flame formation. In addition, the body-force effect on upward or downward propagating flames was investigated, and it was reported that the body-force effect, together with the hydrodynamic effect, has a great influence on the shape of flame fronts [25,26] and on the oscillation of cellular flames [27]. Thus, we need to con-

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sider hydrodynamic and body-force effects in the study of the unstable behavior of cellular-flame fronts.

The author has studied numerically the behavior of laterally moving cellular flames in two- and three-dimensional fields at low Lewis numbers, based on the compressible Navier-Stokes equation [28,29]. The diffusive-thermal effect, and the nonlinear effect of the flame front, play a primary role in the appearance of the lateral movement of cells. Moreover, the body-force effect on the temporal motion of upward propagating flames has been studied [30]. However, a detailed discussion on the lateral movement of cellular flames, and a calculation on downward propagating flames, are lacking.

In this paper, we put the body-force term in the governing equations and perform unsteady calculations of two-dimensional reactive flows at low Lewis numbers. We simulate the evolution of the disturbed flame front to study the body-force effect on the lateral movement of cellular flames.

II. GOVERNING EQUATIONS

We deal with two-dimensional unsteady reactive flows. The assumptions used in this paper are as follows: (i) The chemical reaction is an exothermic one-step irreversible reaction, and the reaction rate obeys the Arrhenius' law. (ii) The burned and unburned gases have the same molecular weights and Lewis numbers, and satisfy the ideal gas equation of state. (iii) The dependence of transport coefficients on temperature is negligible, and the specific heats are constant throughout the whole region. (iv) The radiation, bulk viscosity, Soret effect, Dufour effect, and pressure gradient diffusion are negligible.

The compressible Navier-Stokes equation is employed to take account of the hydrodynamic effect. In the energy equation, the viscous term is disregarded, since its contribution is trivial in the present problem. We use Cartesian coordinates and take the direction tangential to the flame front as the y direction, with the gas velocity in the positive x direction. The body-force effect is assumed to act only in the x direction.

The flow variables in the governing equations are nondimensionalized by the characteristic length, the characteristic velocity, and the density of the unburned gas. The characteristic length is the preheat zone thickness, which is defined as the thermal diffusivity divided by the burning velocity, and the characteristic velocity is the burning velocity. The governing equations of two-dimensional unsteady reactive flows are written in the conservation form as

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=\mathbf{S}, \tag{1}$$

where **U, F, G**, and **S** are vectors given by

$$\mathbf{U}=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ e\\ \rho Y\end{pmatrix},$$

$$\mathbf{F}=\begin{pmatrix}\rho u\\ \rho u^2+\dfrac{p}{\gamma M_0^2}-\mathrm{Pr}\left(\dfrac{4}{3}\dfrac{\partial u}{\partial x}-\dfrac{2}{3}\dfrac{\partial v}{\partial y}\right)\\ \rho u v-\mathrm{Pr}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)\\ (e+p)u-\dfrac{\gamma}{\gamma-1}\dfrac{\partial T}{\partial x}\\ \rho Y u-\dfrac{1}{\mathrm{Le}}\dfrac{\partial Y}{\partial x}\end{pmatrix},$$

$$\mathbf{G}=\begin{pmatrix}\rho v\\ \rho u v-\mathrm{Pr}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)\\ \rho v^2+\dfrac{p}{\gamma M_0^2}-\mathrm{Pr}\left(\dfrac{4}{3}\dfrac{\partial v}{\partial y}-\dfrac{2}{3}\dfrac{\partial u}{\partial x}\right)\\ (e+p)v-\dfrac{\gamma}{\gamma-1}\dfrac{\partial T}{\partial y}\\ \rho Y v-\dfrac{1}{\mathrm{Le}}\dfrac{\partial Y}{\partial y}\end{pmatrix},$$

$$\mathbf{S}=\begin{pmatrix}0\\ G\rho\\ 0\\ QB\rho Y\exp(-E/T)\\ -B\rho Y\exp(-E/T)\end{pmatrix},$$

where t is the time, ρ is the density, u and v are x and y components of the velocity, e is the stored energy, Y is the mass fraction of the unburned gas, p is the pressure, T is the temperature, γ is the ratio of specific heats, M_0 is the Mach number of the burning velocity, Pr is the Prandtl number, Le is the Lewis number, G is the acceleration, Q is the heating value, B is the frequency factor, and E is the activation energy. The equation of state is

$$p=\rho T. \tag{2}$$

III. CALCULATION METHOD

The physical parameters are given to simulate a gas mixtures whose burning velocity is 0.83 m/s and adiabatic flame temperature is 2086 K, where the nondimensional adiabatic flame temperature $T_f=7.0$, at atmospheric pressure and room temperature. The burning velocity is sufficiently small compared with the velocity of sound, $M_0=2.1\times10^{-3}$, so that the nondimensionalized results change very little even though the burning velocity is changed (up to $M_0=3\times10^{-2}$) [31]. Thus, the obtained results are valid for flames with a small burning velocity. The nondimensional parameters used in the present calculation are Pr$=1.0$, $\gamma=1.4$, $Q=21$, and $E=70$. We calculate Le$\leqslant 1$ cases to examine the lateral movement of cellular flames, so that we take Le$=0.5$, 0.6, 0.8, and 1.0. To study the body-force effect, we take $G=-8,-7,-6,...,6$. The $G>0$ ($G<0$) flames correspond to upward (downward) propagating flames. The frequency factor is determined by the condition under which the flame velocity of a plane flame is equal to the set burning velocity.

The initial conditions are provided with the solution of a plane flame, on which we superimpose a sinusoidal disturbance periodic in the y direction. The displacement of the flame front in the x direction due to the disturbance is given by

$$A_0 \sin(2\pi y/\lambda), \tag{3}$$

where A_0 is the initial amplitude and λ is the wavelength. In all cases, we take $A_0=0.8$. We set the wavelength of a disturbance to the linearly most unstable wavelength, i.e., the peculiar wavelength. The reason is that the spacing between cells in a cellular flame is equal to the peculiar wavelength. The peculiar wavelength is obtained from the dispersion relation, which is given by calculations on the evolution of a sufficiently small disturbance. The peculiar wavelengths for Le=0.5, 0.6, 0.8, and 1.0 are 11.5, 12.8, 17.4, and 34.2, respectively [23,32].

The boundary conditions are as follows: In the x direction, except for the inlet velocity, free-flow conditions are used upstream and downstream. The inlet velocity is set to the burning velocity. In the y direction, spatially periodic conditions are used.

The explicit MacCormack scheme [33], which has second-order accuracy in both time and space, is adopted for the present calculation. The computational domain is 80 times the preheat zone thickness in the x direction and one wavelength of the disturbance in the y direction, which is resolved on a 321×65 variably spaced grid. The minimum grid size in the x direction is 1/5. The grid is fine enough to prevent numerical errors from contaminating the solutions. The time-step interval is set to 1×10^{-4} to satisfy the Courant-Friedrichs-Lewy condition.

IV. RESULTS AND DISCUSSION

A sinusoidal disturbance with the peculiar wavelength is superimposed on a plane flame to simulate the cell formation. First, the case of zero acceleration is treated. The flame fronts for Le=0.5, 0.6, and 0.8 are illustrated in Fig. 1. The location of the flame front is defined as the position where the reaction rate takes a maximum value. The unburned gas flows in from the left at the burning velocity, and the burned gas flows out to the right. The superimposed disturbance grows initially with time, and then the front shape becomes cellular. This shape is in qualitative agreement with the experimental results [6,34,35]. Thereafter, the cellular flame moves upstream, indicating that the flame velocity is increased. The increment in flame velocity is equal to the moving rate of the cellular flame, since the inlet velocity is set to the burning velocity. The flame-velocity increments for Le=0.5, 0.6, and 0.8 are 1.08, 0.64, and 0.30, respectively. After the cell formation, the cellular flame begins to move in the y direction. This lateral movement is due to the overshoot

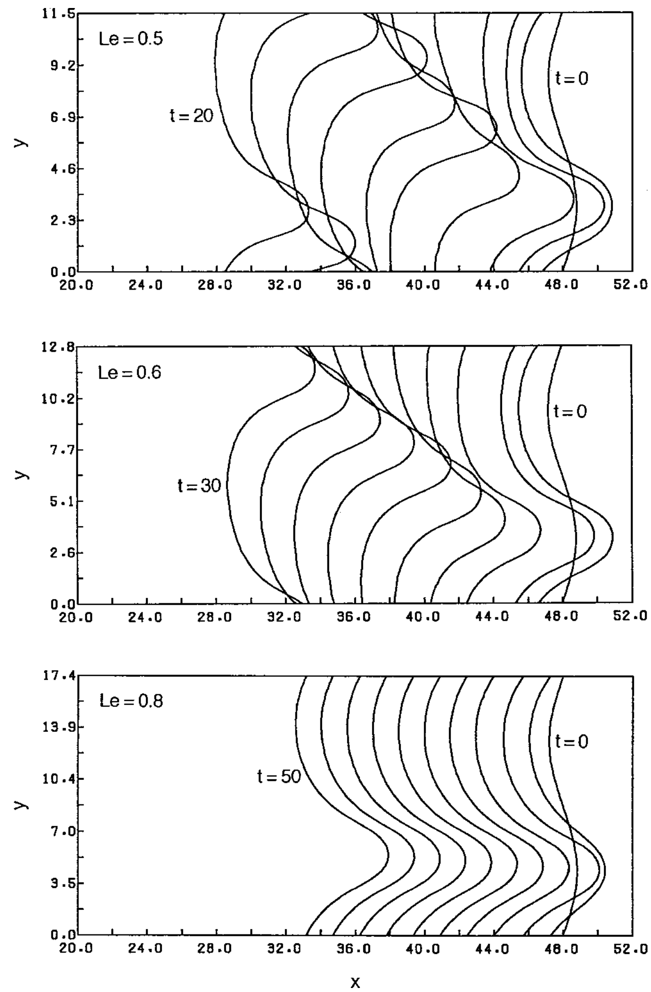

FIG. 1. Flame fronts for $G=0$, Le=0.5, 0.6, and 0.8 ($t=0,2,4,...,20$ at Le=0.5; $t=0,3,6,...,30$ at Le=0.6; $t=0,5,10,...,50$ at Le=0.8).

of temperature generated by the diffusive-thermal effect at low Lewis numbers and by the nonlinear effect of the flame front [28]. The lower the Lewis number, i.e., the higher the instability level, the larger the lateral velocity of cells. The lateral velocities V_L for Le=0.5, 0.6, and 0.8 are 0.83, 0.41, and 0.024, respectively.

Next, the body-force effect on the lateral movement of cellular flames is studied. We take Le=0.5. As for upward (downward) propagating flames, the acceleration G is positive (negative) and the body-force effect has a destabilizing (stabilizing) influence. The flame fronts for $G=0$, 2, and 4 ($\geqslant 0$) are illustrated in Fig. 2(a). The superimposed disturbance evolves, and a cellular flame is formed owing to intrinsic instability. As the acceleration increases, the lateral velocity of cells decreases, even though the instability level becomes higher. The lateral velocities for $G=0$, 2, and 4 are 0.83, 0.59, and 0.30, respectively. In addition, the cell depth increases with an increase in acceleration, since the instability of premixed flames becomes stronger. The cell depths for $G=0$, 2, and 4 are 5.61, 5.91, and 6.58, respectively. For downward propagating flames, on the other hand, we have interesting results on the lateral velocity. The flame fronts for

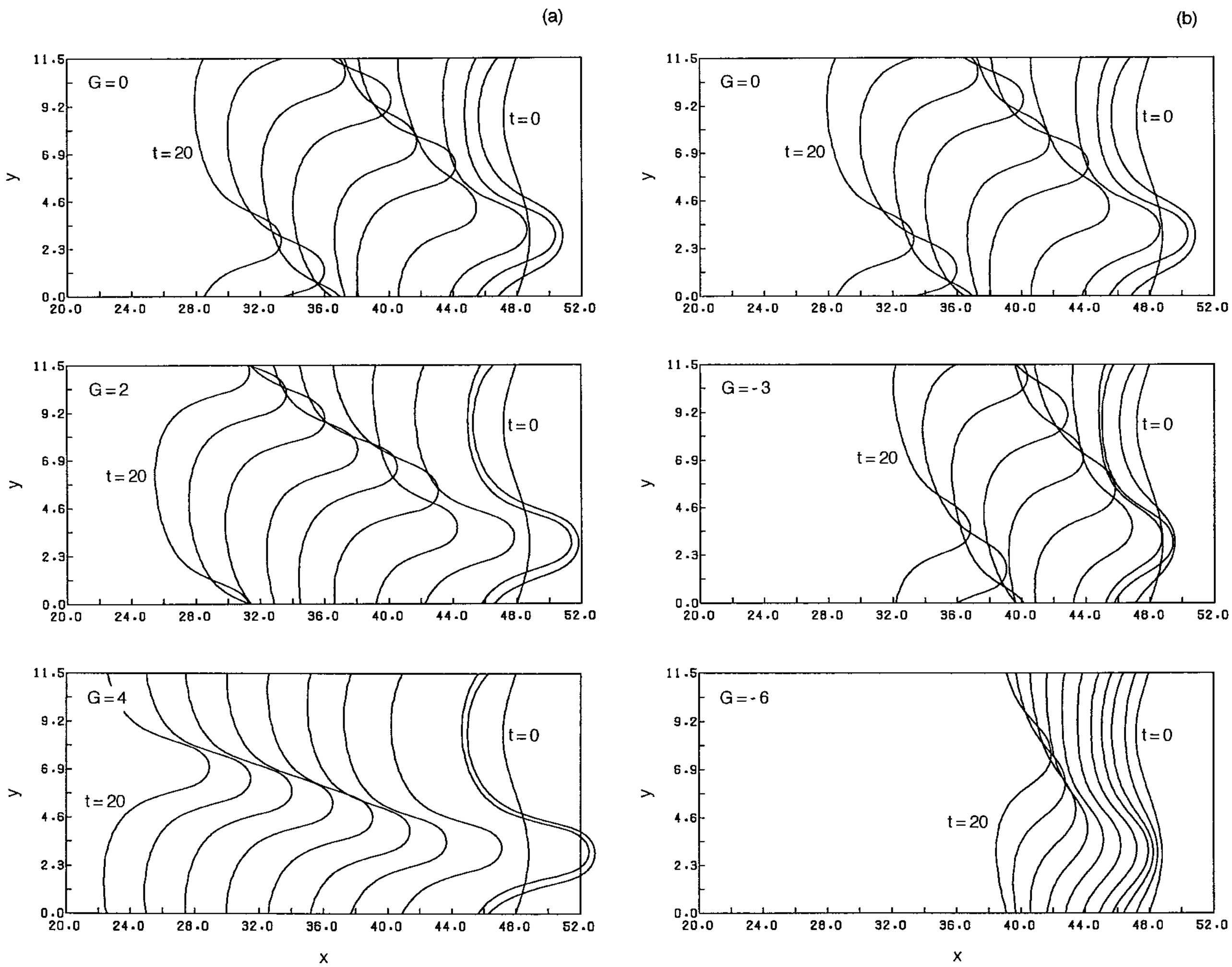

FIG. 2. Flame fronts for Le=0.5 ($t=0,2,4,...,20$): (a) $G=0$, 2, and 4; (b) $G=0$, -3, and -6.

$G=0$, -3, and -6 ($\leqslant 0$) are illustrated in Fig. 2(b). The lateral velocity of the $G=-3$ flame is 1.00, which is larger than that of the $G=0$ flame (= 0.83). However, the lateral velocity decreases ($V_L=0.66$ at $G=-6$) by increasing the absolute value of acceleration further. At $G\leqslant -9$ no cellular flame is formed, since the body-force effect prevents hydrodynamic and diffusive-thermal effects from destabilizing the flame front. The dependence of lateral velocity on the acceleration is not explicated only by the change in instability level.

We take Le=0.6. The flame fronts for $G\geqslant 0$ and $G\leqslant 0$ are illustrated in Fig. 3. Cellular flames are formed at $G\geqslant -5$, and cells move laterally. The relation between the lateral velocity and the acceleration is similar to that of the Le=0.5 flame. The lateral velocities for $G=0$, 1, 2, -2, and -4 are 0.41, 0.22, 0.08, 0.62, and 0.22, respectively. These values are smaller than those of the Le=0.5 flame, since the instability level due to the diffusive-thermal effect is lower. In addition, the cell depth increases (decreases) as the absolute value of acceleration increases at $G>0(G<0)$, just as the Le=0.5 flame did.

We take the Lewis number higher, i.e., Le=0.8 and 1.0. For Le=0.8, the laterally moving cellular flame appears at $G\geqslant -2$, just as Le=0.5 and 0.6 flames did. The lateral velocities for $G=0$, 1, 2, -1, and -2 are 0.024, 0.016, 0.012, 0.024, and 0.016, respectively. Compared with the results of Le=0.5 and 0.6 flames, the lateral velocity is considerably small, but its dependence on the acceleration is consistent. For Le=1.0, on the other hand, no lateral movement of cells is observed, which is in agreement with the results of other calculations [36]. The reason is that the diffusive-thermal effect, which plays a primary role in the appearance of the lateral movement of cells, does not occur in this flame [24].

Figure 4 shows the lateral velocities of cells for Le =0.5, 0.6, and 0.8, depending on the acceleration. The lower the Lewis number, the larger the lateral velocity, which is due to the increase in instability level caused by the diffusive-thermal effect. When flames are propagated upward ($G>0$), the lateral velocity decreases as the acceleration increases, even though intrinsic instability becomes stronger. When flames are propagated downward ($G<0$), on the other hand, the lateral velocity takes a maximum value at the specific acceleration and decreases by increasing $|G|$.

To investigate the mechanism of the dependence of lateral

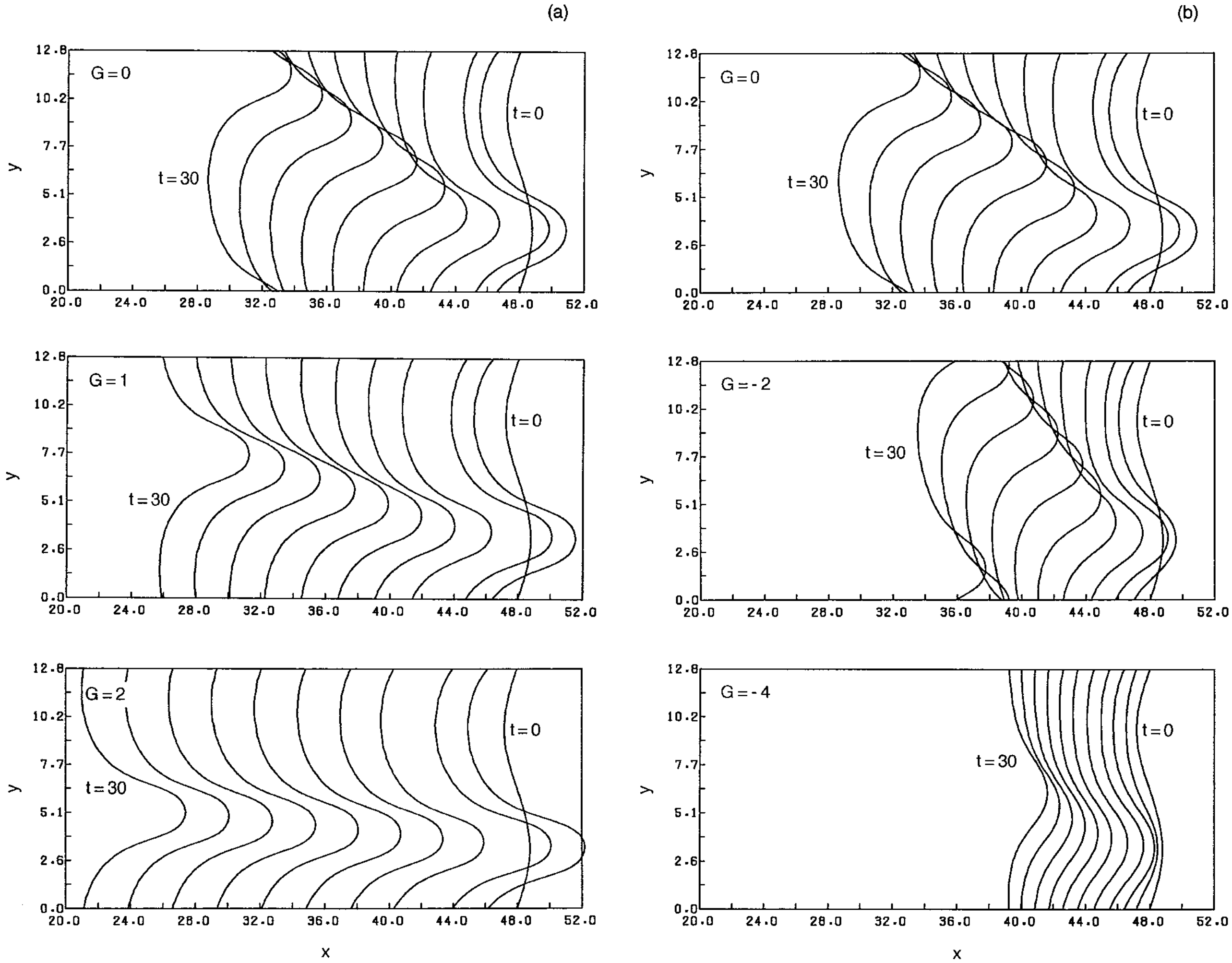

FIG. 3. Flame fronts for Le = 0.6 (t = 0,3,6,...,30): (a) G = 0, 1, and 2; (b) G = 0, −2, and −4.

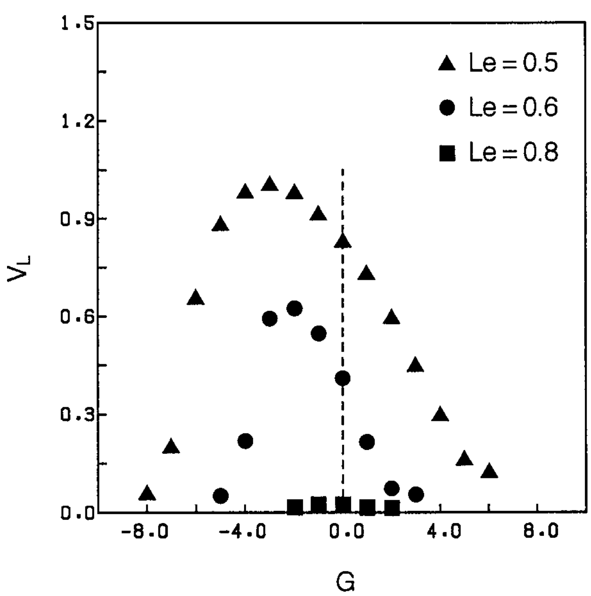

FIG. 4. Lateral velocities of cells for Le = 0.5, 0.6, and 0.8, depending on the acceleration.

velocity on the acceleration, the temperature distributions at a convex flame front with respect to the unburned gas for Le = 0.5 are shown in Fig. 5. The flame front is convex at y = 9.4, 1.6, 10.4, and 3.0 for G = 0, 4, −3, and −6, respectively. The temperature has an overshoot near the flame front, which is observed in Le $<$ 1 flames. The overshoot of temperature is the origin of the lateral movement of cells, i.e., it causes a breaking of the reflection symmetry of cells and then the lateral movement is introduced with their unsymmetrical shape [18,28]. The lateral velocity of cells is affected by the maximum flame temperature, since the temperature distribution is the most significant factor in the appearance of lateral movement. The maximum flame temperatures for G = 0, 4, −3, and −6 are 7.74, 7.79, 7.72, and 7.55 ($>T_f = 7.0$), respectively. We have a high-temperature region behind a convex flame front, which also has an influence on the lateral velocity. The breadths of high-temperature region in the x direction for G = 0, 4, −3, and −6 are 2.2, 3.5, 1.4, and 0.7, respectively. Variations in maximum flame temperature and in breadth of high-temperature region are due to the change in cell depth. The cell depths for G = 0, 4, −3, and −6 are 5.61, 6.58, 4.93, and 3.32, respectively.

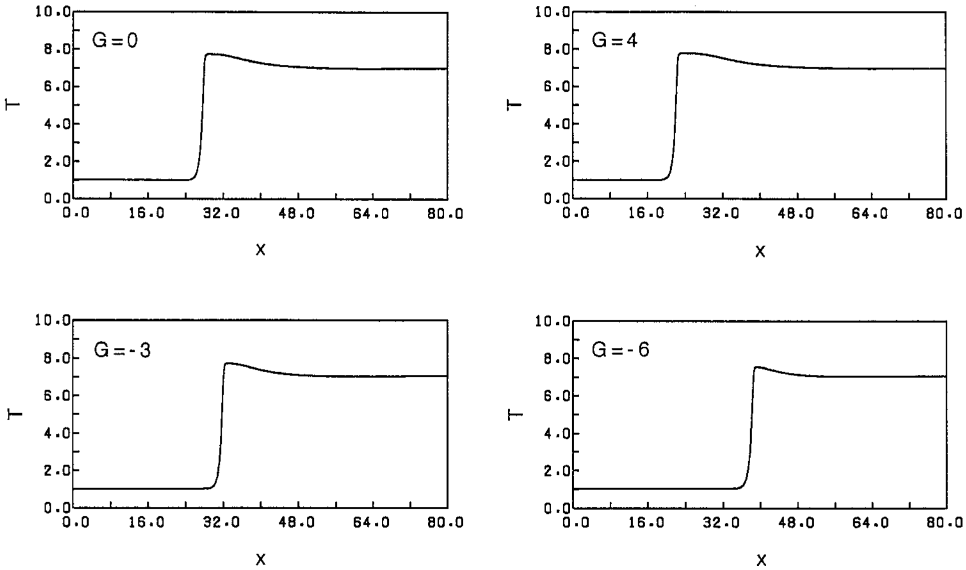

FIG. 5. Temperature distributions for Le=0.5, $G=0$, 4, -3, and -6 ($t=20$), where the flame front is convex with respect to the unburned gas.

The overshoot of temperature induces the unstable behavior of cellular flames, and the instability level on lateral movement depends on the temperature distribution. The augmentation in maximum flame temperature raises the instability level on lateral movement, and the increase in high-temperature region reduces it. In the $G=4$ ($G=-3$) flame, the maximum flame temperature is slightly higher (lower) than that of the $G=0$ flame, and high-temperature region is considerably broader (narrower). The decrease (increase) in instability level due to the broadness (narrowness) of the high-temperature region exceeds the increase (decrease) due to the augmentation (diminution) in maximum flame temperature. Thus, the lateral velocity is smaller (larger) than that of the $G=0$ flame. The maximum flame temperature for $G=-6$ is considerably lower. Thus, the instability level on lateral movement becomes lower, even though high-temperature region is narrower, and then the lateral velocity becomes smaller. The dependence of lateral velocity on the acceleration is due to the augmentation and diminution in maximum flame temperature and to the broadness and narrowness of high-temperature region.

V. CONCLUDING REMARKS

We have performed unsteady calculations of two-dimensional reactive flows to study the body-force effect on the lateral movement of cellular flames at Lewis numbers lower than unity. The body-force effect has a great influence on the lateral velocity of cells. When the acceleration is positive, i.e., flames are propagated upward, the lateral velocity decreases as the acceleration increases. When the acceleration is negative, on the other hand, the lateral velocity takes a maximum value at the specific acceleration and decreases by increasing the absolute value of acceleration. The dependence of lateral velocity on the acceleration is due to the augmentation and diminution in maximum flame temperature and to the broadness and narrowness of high-temperature region behind a convex flame front.

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